ATTENUATION OF STRONG SHOCK WAVES IN TWO-PHASE AND HETEROGENEOUS MEDIA

L. V. Al'tshuler and B. S. Kruglikov

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1. General Laws of the Attenuation of Shock Waves. In many of the uses of explosive processes it is important to know the characteristics of diverging shock waves at various distances from the seat of the energy release. The greatest decrease of pressure and the most effective localization of the mechanical effects of a strong explosion are produced in media which have a small thermal elasticity (small Grüneisen parameter) and a high compressibility. For a given energy of the explosion the first of these factors determines the peak pressure in the near zone, and the second determines the rate of attenuation of the shock wave with distance.

There are three causes of shock wave attenuation [1]: the interaction of the discontinuity with the overtaking unloading waves, the geometrical divergence of the waves, and relaxation processes. According to [1] the general formula for the attenuation caused by the first two factors was derived in 1942-1943 by Harris for spherical cylindrical, and plane waves. For the more general case of a wave surface of arbitrary configuration, characterized by two Gaussian radii of curvature R_1 and R_2 and the normal N, the equation

$$D\left[(D-u)+\rho c^2\left(\frac{du}{dp}\right)\right]\frac{dp}{dN}=[(D-u)^2-c^2]\left(\frac{\partial p}{\partial N}\right)_f-\rho c^2 u(D-u)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \quad (1.1)$$

holds, where D is the velocity of the shock front (SF), p, u, and ρ are respectively the pressure, mass velocity, and density of the medium at the SF, c is the speed of sound, dp/dN is the derivative determining the attenuation of the shock discontinuity, and $(\partial p/\partial N)_f$ characterizes the gradient of the pressure drop across the SF.

For spherical (n = 3, $R_1 = R_2 = R$), cylindrical (n = 2, $R_1^{-1} = 0$, $R_2 = R$), and plane (n = 1, $R_1^{-1} = R_2^{-1} = 0$) waves, Eq. (1.1) can be written in the compact form

$$\left(\frac{\rho}{\rho_0}\right)\frac{M^2(D+uD'_u)+D}{D+uD'_u}\frac{d\ln p}{d\ln R}=-(1-M^2)\left(\frac{\partial\ln p}{\partial\ln R}\right)_f-(n-1), \quad (1.2)$$

where M = (D − u)/c is the Mach number, and ρ_0 is the initial density of the medium.

For strong shock waves (ρ/ρ_0 = const = h) the adiabat in (p, u) coordinates has a tangent D'_u = D/u which passes through the origin. In this case (1.2) becomes

$$h(M^2+0.5)\frac{d\ln p}{d\ln R}=-(1-M^2)\left(\frac{\partial\ln p}{\partial\ln R}\right)-(n-1). \quad (1.3)$$

We use (1.3) to examine the attenuation of a strong shock wave in an ideal gas, in a porous incompressible Kompaneits medium, and in an equilibrium two-phase mixture. The analysis showed that it is expedient to investigate the attenuation of shock waves in special skeleton systems which are modeled by a heterogeneous medium based on a two-velocity two-temperature model [2].

Explosion in an Ideal Gas. The solution of the problem of a strong explosion in an ideal gas found in [3] leads to familiar relations which show that the law of attenuation of the shock wave parameters at the front is described in different geometries (n = 1, 2, 3) by the following relations for the pressure, the mass velocity, and the specific momentum:

$$\frac{d\ln p}{d\ln R}=-n,\quad \frac{d\ln u}{d\ln R}=-\frac{n}{2},\quad \frac{d\ln I}{d\ln R}=-\frac{n}{2}+1. \quad (2.1)$$

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The specific momentum I is defined here as the momentum $I = R^{-(n-1)}\int_0^R \rho(r)u(r)r^{n-1}dr$, contained in the angular sector which intercepts a unit area of the wave front.

For a spherical point explosion with an energy release E in a gas with a constant adiabatic exponent γ and the limiting compression $h = (\gamma + 1)/(\gamma - 1)$, the density, pressure, mass velocity, and specific momentum at the SF are given by the expressions

$$\rho = h(\gamma)\rho_0, \quad p = q_1(\gamma)\left(\frac{E}{R^3}\right), \quad u = q_2(\gamma)\left(\frac{E}{\rho_0 R^3}\right)^{1/2}, \qquad (2.2)$$
$$I = q_3(\gamma)\left(\frac{E\rho_0}{R}\right)^{1/2}$$

where the coefficients $q_1(\gamma) = (8/25)[1/(\alpha(\gamma)(\gamma+1))]$ and $q_2(\gamma) = (4/5)[1/((\gamma+1)^2\alpha(\gamma))]^{1/2}$ depend on γ [3], and $\alpha(\gamma)$ is the constant factor in the Sedov solution. The coefficient $q_3(\gamma)$ can be calculated numerically from the known self-similar solution [3, 4].

The values of the coefficients in (2.2) are listed in Table 1 for various values of γ. A comparison of $q_1(\gamma)$ and the Grüneisen parameter $\Gamma = \gamma - 1$ (column 4 of Table 1) shows that they are approximately proportional, which leads to an almost linear dependence of the pressure at the SF on the thermal elasticity Γ of the medium. The magnitude of the specific momentum is proportional to $\rho_0^{1/2}$, and, according to the data of Table 1, is a monotonically increasing function of γ.

For an explosion in a closed volume we are considering, the loss of momentum from the decrease of the initial density of the medium (decoupling [5]) occurs as a result of a contraction in the time scale of the wave profile, and, accordingly, of a decrease in the time of action of the pressures on the wall of the closed volume. We illustrated this by using the finite-difference method [6] to calculate the effect of an explosion ($E = 7.1 \times 10^{12}$ J) in a gas with a density of 1 kg/m^3 (ordinary air) and in rarefied air ($\rho_0 = 0.1$ kg/m^3) in a closed volume of radius R = 40 m. Figure 1 shows the time dependence of the pressure on the wall of a closed volume for the first (1) and second (2) cases. It can be seen that the repeated action of circulating waves on the average is near the static pressure of a uniform distribution of the energy of the explosion (dashed line).

The expression for the gradient of the pressure drop across the SF for an arbitrary γ was determined by comparing (1.3) and (2.1). The result is the relation

$$\left(\frac{\partial \ln p}{\partial \ln R}\right)_f = \frac{n(3\gamma - 1) + 2\gamma(\gamma - 1)}{\gamma^2 - 1}. \qquad (2.3)$$

It follows from (2.3) that the spatial width of the pressure peak does not depend on density, but with a decrease in γ the profile is compressed, the pressure gradient is increased, and becomes infinite as $\gamma \to 1$. The analytic expression derived is a direct consequence of the theory of a self-similar explosion [3], but is not cited in the corresponding monographs.

3. Porous Incompressible Kompaneits Medium (PIM). The equation of state and the shock adiabat in a PIM in (p, V) coordinates are represented by a vertical line at a distance V_0/h from the origin [7], where h is the compaction parameter, equal to the ratio of the initial and final volumes V_0/V. Behind the SF the speed of sound in the compressed state is infinite, and M = 0. For a PIM Eq. (1.3) takes the form

$$\frac{h}{2}\frac{d\ln p}{d\ln R} = -\left(\frac{\partial \ln p}{\partial \ln R}\right)_f - (n - 1). \qquad (3.1)$$

For large h in a self-similar wave which has traversed a distance R at the inner boundary of the compacted layer, the pressure is zero. Then the pressure gradient inside a layer of thickness R/h is constant and equal to nhp/R. If h >> n − 1, we find from (3.1) that the relation

$$d\ln p/d\ln R = -2n, \; d\ln u/d\ln R = -n, \qquad (3.2)$$
$$d\ln I/d\ln R = -n + 1$$

TABLE 1

γ	$\alpha(\gamma)$	$q_1(\gamma)$	$q_1(\gamma)/(\gamma-1)$	$q_2(\gamma)$	$q_3(\gamma)$	γ	$\alpha(\gamma)$	$q_1(\gamma)$	$q_1(\gamma)/(\gamma-1)$	$q_2(\gamma)$	$q_3(\gamma)$
1,1	3,4195	0,0446	0,446	0,2060	0,064	1,3	1,1436	0,1217	0,406	0,3253	0,095
1,15	2,2894	0,0650	0,433	0,2459	0,076	1,4	0,8510	0,1567	0,392	0,3613	0,106
1,2	1,7198	0,0846	0,423	0,2773	0,084	5/3	0,4936	0,2431	0,365	0,4270	0,120

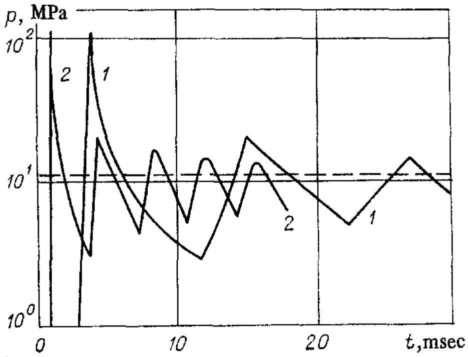

Fig. 1

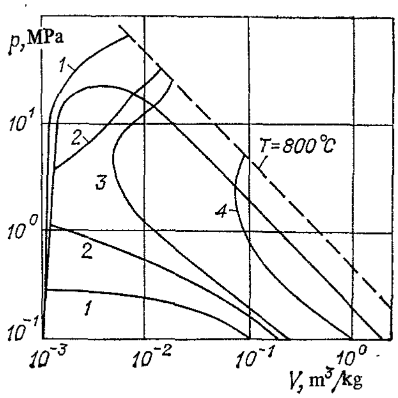

Fig. 2

is fairly accurately satisfied. It follows from (3.2) that the pressures, velocities, and momenta are damped very much more rapidly in a PIM with a large h than in an ideal gas. In particular, for a spherical shock wave we have instead of (2.2)

$$p \sim 1/R^6,\ u \sim 1/R^3,\ I \sim 1/R^2. \tag{3.3}$$

Relations (3.3) can be used to describe the results of experiments on explosions in media such as snow, vermiculite, water foam, etc. In particular, Eq. (3.3) naturally accounts for the empirical relation found in [8] for the decay of momentum in a foamy medium. Questions of the attenuation of shock waves in foamy media were studied in detail experimentally and theoretically in [8-10].

4. Equilibrium Vapor—Liquid Medium. Let us consider an equilibrium two-phase mixture of water and water vapor. Adiabats calculated along the equilibrium curve of water [11] are shown in Fig. 2. A similar form of the adiabats was obtained in [12, 13] from equations of state of water developed by the authors of these papers. Adiabats 1-4 correspond to initial densities of the mixture ρ_0 = 10, 5, 4, and 1 kg/m³. Adiabats of type 1 correspond to a preponderant weight content of water, and are characterized by an anomalously small Grüneisen parameter ($\Gamma \sim 0.02$) and a very large value of the limiting compression $h \sim 100$ ($h = 1 + 2/\Gamma$). Such behavior of a medium is due physically to condensation of vapor in the shock wave, which decreases the Grüneisen ratio. For adiabats of type 4 with a preponderant vapor content there is a smaller decrease of the Grüneisen parameter as a result of the absorption of the latent heat of vaporization, in spite of some increase during the evaporation of the molar content of the gaseous phase. On the vertical branch of adiabat 1 $D \sim c$, and $D - u = D/h$. Such values of $D - u$ and c lead to vanishingly small Mach numbers.

As a result, for certain initial densities the attenuation of strong shock waves in equilibrium two-phase mixtures at pressures below the critical value p_c can be described by the PIM model.

Actually, three flow regions develop for an explosion in such a two-phase medium: the near zone where $p \gg p_c$ and the material behaves like an ideal gas; the intermediate layer where the pressure is decreased to $p < p_c$; the outer layer in which the PIM model is realized, i.e., Eqs. (3.3) are satisfied in the spherical case.

Figure 3 shows some results for a two-phase vapor—liquid lithium mixture with a type 4 adiabat calculated by the method of [6] using a model equation of state constructed by the authors. Curve 1 characterizes the variation of pressure at the SF in an ideal gas (γ = 1.4) for a point explosion ($E = 0.84 \times 10^{12}$ J). Curve 2 determines a substantially stronger attenuation in the two-phase lithium medium (ρ_0 = 1 kg/m³) located in the concentric region $R \geq 10$ m.

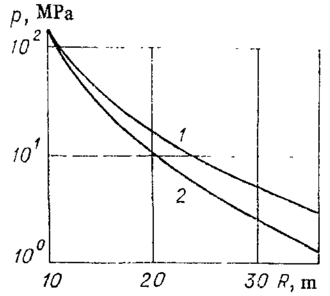

Fig. 3

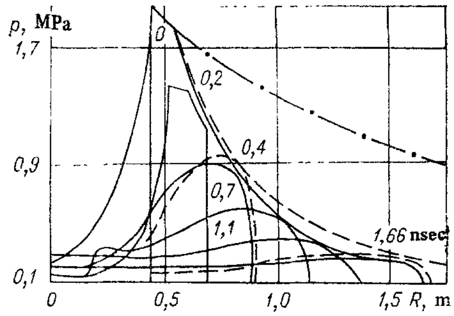

Fig. 4

5. Attenuation of Shock Waves in a Stationary Skeleton System. The formation of internal superheated regions decreases the attenuating properties of the porous incompressible and two-phase media considered above. Substantial attenuation of a shock wave is achieved by introducing into its path a region containing a streamlined stationary skeleton which elongates the SF and transforms the one-dimensional gas flow into a system of jets which are slowed down in their encounter with the elements of the skeleton.

In the first approximation such a system is modeled mathematically by a process of penetration of a shock wave into a heterogeneous air-suspended layer with stationary ("frozen") particles, which is naturally described by the two-velocity two-temperature model [2]. Calculations were performed by the method of coarse particles [14].

The conventional assumptions about the mechanics of multiphase media were assumed to hold for the air-supported layer: The particles are spherical and monodisperse; the distances between particles are small in comparison with the characteristic scales of flows; the effects of viscosity and heat conduction are important only in the interaction of the phases; there are no collisions between particles, which are not deformed or crushed. We solve the following problem to illustrate the attenuating action of a skeleton system. A plane wave with a triangular velocity profile, a wavelength of 0.45 m, M = 4.17, and a maximum pressure of 2 MPa (counterpressure 0.1 MPa) is incident on an air-supported layer of stationary iron particles 1.5 m thick in front of an obstacle. The particle diameter d was varied from 60 to 1200 μm, and the volume content of the particles α_V from 0.1 to 6%, which corresponds to a variation of the lattice parameter d/l of the system from 8 to 2, where l is the distance between particle centers. We studied the variation of the parameters in an advancing shock wave and in one reflected from a rigid wall, including the pulsed effect. We showed that the attenuation of a shock wave increases with an increase in the concentration and a decrease in the size of the particles. Figure 4 shows pressure profiles in the advancing wave at various times for particles with d = 60 μm for $\alpha_V = 0.1\%$ (the dashed curves are profiles for moving particles), and also the variation of the peak pressure of the shock wave with distance for an ideal gas without particles ($\gamma = 1.4$) (dash-dot curve), moving (dashed curve), and stationary (solid curve) particles. There is very strong damping of the pressure in the air-supported layer (the cases of moving and stationary particles are negligibly different from one another). The pressure in the shock wave reflected from the wall is decreased by a factor of 11, and the magnitude of the maximum impulse of the express pressure is smaller by a factor of 4 than in a pure gas. Nearly the same result is obtained for a simultaneous increase of the particle size and volume content by an order of magnitude.

6. Skeleton System with an Evaporating Component. The mechanical energy of an explosion is completely absorbed by introducing into a skeleton system an evaporating component in the form of drops, foam, or aerosols with a large heat of vaporization of the particles.

For a latent heat of vaporization θ the minimum necessary mass m of the evaporating component for an explosion of energy E is given approximately by the relation

$$m = E/\theta. \qquad (6.1)$$

Numerical estimates based on (6.1) show that $m \approx 0.07E$ for a graphite (carbon) aerosol, $m \approx 0.2E$ for lithium, and $m \approx 2E$ for water, where E is in tons of TNT equivalent, and m is in tons.

The use of a skeleton system with evaporating components can ensure an effective solution of a number of practically important problems such as the creation of technological explosion chambers with minimum wall thicknesses, the shielding of pulsed reactors against mechanical and radiation damage [15], the blocking of the propagation of detonation waves in rock workings [16], etc.

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